GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION 1

HSC ASSESSMENT

TASK 1 2025

(December 2024)

Student Number:_____.

		ME12-1	ME12-2	ME12-3	ME12-4	ME12-5	ME12-6	ME12-7
1-5	**/5**			√				√
6(a)	**/3**							√
6(b)(c)	**/6**			√				√
7	**/10**			√				√
8	**/12**			√				√
9	**/9**			√				√
10	**/13**	√						√
11	**/8**			√				√
TOTALS	**/66**	**/13**		**/50**				**/66**

Year 12 Mathematics Extension 1 outcomes

A student:

ME12-1 applies techniques involving proof or calculus to model and solve problems

ME12-2 applies concepts and techniques involving vectors and projectiles to solve problems

ME12-3 applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations

ME12-4 uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution

ME12-5 applies appropriate statistical processes to present, analyse and interpret data

ME12-6 chooses and uses appropriate technology to solve problems in a range of contexts

ME12-7 evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

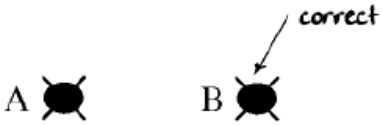
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.



Select the alternative A, B, C or D that best answers the question.

1.	A ○	B. ○	C. ○	D. ○
2.	A ○	B. ○	C. ○	D. ○
3.	A ○	B. ○	C. ○	D. ○
4.	A ○	B. ○	C. ○	D. ○
5.	A ○	B. ○	C. ○	D. ○

GIRRAWEEN HIGH SCHOOL

HSC TASK 1 2025 *(December 2024)*

MATHEMATICS

EXTENSION 1

Time allowed – 90 Minutes

(plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- NESA-approved calculators may be used. Reference sheets will be provided.
- Each question attempted is to be marked clearly Question 6, Question 7 etc. Each question is to be answered on a new page in your answer booklet.
- You may ask for spare answer booklets if you need them.
- For Multiple choice: Fill in the circle corresponding to the correct answer on the multiple choice answer sheet in your answer booklet.
- For Questions 6-11: All necessary working should be shown in each question. Marks may be deducted for careless or badly arranged work.

Multiple Choice begins on the following page

For Multiple choice Questions 1-5: Fill in the circle corresponding to the correct answer on the multiple choice answer sheet in your answer booklet.

Question 1

Daniel was solving $\sin 2x = cosx, 0 \leq x \leq 2\pi$. His working was:

$\sin 2x = \cos x$ Line 1

$2 \sin x \cos x = \cos x$ Line 2

$\sin x = \frac{1}{2}$ Line 3

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ Line 4

Daniel has made a mistake on one of the lines. The line he made a mistake on was

(A) Line 1

(B) Line 2

(C) Line 3

(D) Line 4

Question 2

$2 \sin 2x \cos x$ can be rewritten using products to sums as

(A) $\sin 3x + \sin x$

(B) $\cos 3x + \cos x$

(C) $\sin 3x - \sin x$

(D) $\cos 3x - \cos x$

Question 3

If $2 \sin \theta - 3 \cos \theta$ is arranged in the form $Rcos(\theta + \alpha)$ then $\sin \alpha =$

(A) $\frac{2}{\sqrt{13}}$

(B) $-\frac{2}{\sqrt{13}}$

(C) $\frac{3}{\sqrt{13}}$

(D) $-\frac{3}{\sqrt{13}}$

Multiple choice continues on the following page

Question 4

$2\sin\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-x\right)=$

(A) $\cos 2x$

(B) $\sin 2x$

(C) $2\cos x$

(D) $2\sin x$

Question 5

If $t=\tan\frac{x}{2}$ then $\tan x+\sec x=$

(A) $\frac{2t+1-t^2}{1+t^2}$

(B) $\frac{1}{t}$

(C) $\frac{1+t}{1-t}$

(D) $\frac{1+t^2}{1-t^2}$

Examination continues on the following page

Question 6 ***(9 marks) (Start solutions on a new page)*** **Marks**

(a) Sketch the region in the number plane where the inequalities $x^2 + y^2 \leq 25 \; and \; 4x - 3y > 0$ hold simultaneously. **3**

(b) Use compound angle formulae to find the exact value of

(i) $\sin 130^o \cos 70^o - \cos 130^o \sin 70^o$ **2**

(ii) $\frac{\tan 110^o - \tan 80^o}{1 + \tan 110^o \tan 80^o}$ **2**

(c) Show that the exact value of $cos \frac{\pi}{12}$ is $\frac{\sqrt{6}+\sqrt{2}}{4}$. **2**

Question 7 ***(10 Marks) (Start solutions on a new page)***

(a) (i) Express $\sqrt{3}\cos\theta - \sin\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and α is in radians. **3**

(ii) Hence solve $\sqrt{3}\cos\theta - \sin\theta = 1 \; for \; 0 \leq \theta \leq 2\pi$. **2**

(b) Using the auxiliary angle method and choosing a substitution so that $0 < \alpha < 90^o$, solve $3\sin\theta - 2\cos\theta = 1$ for $0 \leq \theta \leq 360^o$. **5**

Examination continues on the following page

Question 8 ***(12 marks) (Start solutions on a new page)*** **Marks**

(a) Using the substitution $t = \tan\frac{\theta}{2}$, solve the equation **5**

$3\sin\theta + 4\cos\theta = 2 \; for \; 0 \leq \theta \leq 360^o$.

(b) (i) Express $\sqrt{2}\cos x + \sqrt{2}\sin x$ in the form $R\sin(x + \alpha)$. **4**

(ii) Hence sketch $y = \sqrt{2}\cos x + \sqrt{2}\sin x \; for \; 0 \leq x \leq 2\pi$. **3**

Question 9 ***(9 Marks) (Start solutions on a new page)***

(a) Solve for $x, 0 \leq x \leq 360^o$: **4**

$\cos 2x - \sin x = \frac{1}{2}$

(b) (i) Show using sums to products formulae that **2**

$\sin 8\theta + \sin 6\theta - \sin 4\theta - \sin 2\theta = 4\,cos\,5\theta\,sin\,2\theta\,cos\,\theta$

(ii) Hence solve for $\theta, 0 \leq \theta \leq 2\pi$ **3**

$\sin 8\theta + \sin 6\theta - \sin 4\theta - \sin 2\theta = 0$

Question 10 ***(13 marks) (Start solutions on a new page)***

(a) Prove by mathematical induction for all integers $n \geq 1$ **4**

$1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + n(n+1) = \frac{1}{3}n(n+1)(n+2)$

(b) Prove by mathematical induction for all integers $n \geq 1$: **4**

$5^{2n} - 3^{2n}$ is divisible by 16.

(c) Prove by mathematical induction for all integers $n \geq 1$: **5**

$$\frac{1}{1\times2\times3} + \frac{1}{2\times3\times4} + \cdots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

Examination continues on the following page

Question 11 ***(8 marks) (Start solutions on a new page)*** **Marks**

(i) Prove that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$ **3**

(ii) Use sums to products formulae to show that **2**

$$\cos\frac{4\pi}{9} + \cos\frac{2\pi}{9} = \cos\frac{\pi}{9}$$

(iii) Hence use (i) and (ii) to show that $\cos\frac{\pi}{9}$ is a solution to **3**

the equation $8x^3 - 6x - 1 = 0$

END OF EXAMINATION

Mathematics Assessment Solutions

Girraween High School

Page 1

☐ Year 11	☑ Extension 1	Year: 2025
☐ Year 12	☐ Extension 2	Task Number 1

Suggested solutions	Marker's Feedback

Multiple Choice:

(1) (C) Line 3 was where the error was.
Dividing by $\cos x$ eliminated solutions where $\cos x = 0 \rightarrow$ namely $x = \frac{\pi}{2}$ & $x = \frac{3\pi}{2}$.

(2) (A) As $\sin A\cos B = \frac{1}{2}[\sin(A+B)+\sin(A-B)]$

$2\sin x\cos x = \sin(2x+x)+\sin(2x-x)$
$= \sin 3x + \sin x.$

(3) $2\sin\theta - 3\cos\theta \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$
(B) Equating parts

$2\sin\theta = -R\sin\theta\sin\alpha$ | $-3\cos\theta = R\cos\theta\cos\alpha$
$-2 = R\sin\alpha$ (1) | $-3 = R\cos\alpha$ (2)

$(1)^2 + (2)^2 : R^2(\cos^2\alpha + \sin^2\alpha) = 2^2 + 3^2$.
$R = \sqrt{13}$.

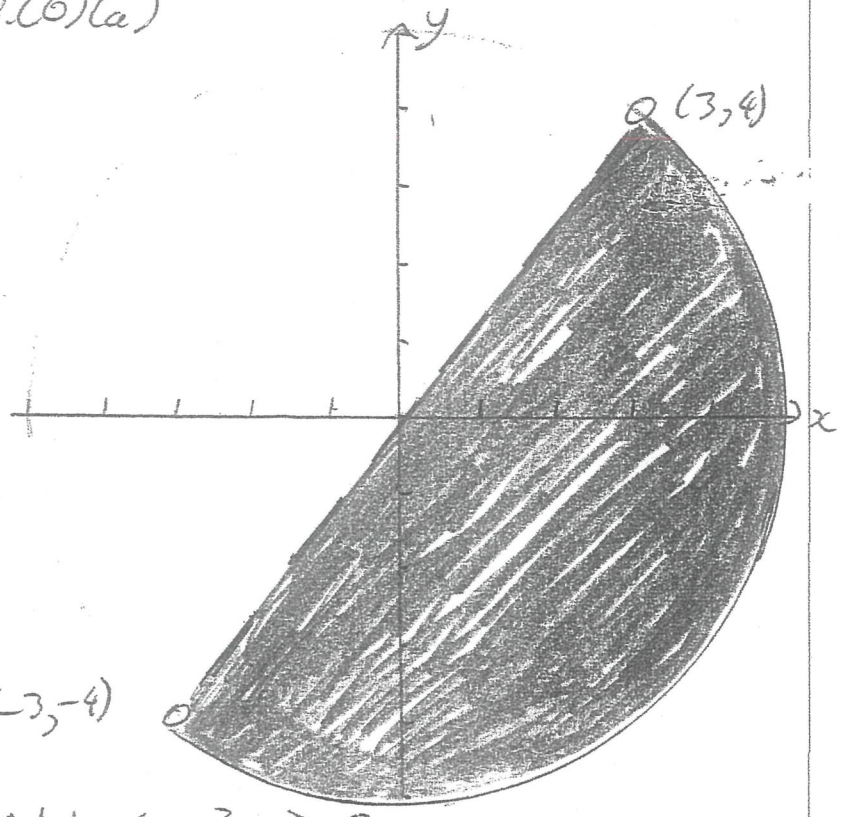
sub. $R = \sqrt{13}$ in (1) $-2 = \sqrt{13}\sin\alpha$
$\frac{-2}{\sqrt{13}} = \sin\alpha$.

(4) $2\sin\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-x\right)$
(A) $= \sin\left(\frac{\pi}{2}-2x\right)$
$= \cos 2x$

(5) $\tan x + \sec x$
(C) $= \frac{2t}{1-t^2} + \frac{1+t^2}{1-t^2}$
$= \frac{(1+t)^2}{(1-t)(1+t)}$
$= \frac{1+t}{1-t}$.

Suggested solutions	Marker's Feedback

Q.(6)(a)



Note: $4x-3y>0$

$4x>3y$

$\frac{4}{3}>y$ or $y<\frac{4}{3}x$

1 — open circles

1 — solid, dotted lines

1 — correct region

Very few students are required to revise region otherwise mostly done well. Some students forgot open circles and some dotted & solid lines.

(b)(i) $\sin 130^\circ\cos 70^\circ-\cos 130^\circ\sin 70^\circ$

$=\sin(130^\circ-70^\circ)$

$=\sin 60^\circ$

$=\frac{\sqrt{3}}{2}$

1 — $\sin(130^\circ-70^\circ)$

1 — $\frac{\sqrt{3}}{2}$.

Mostly done well!

(ii) $\frac{\tan 110^\circ-\tan 80^\circ}{1+\tan 110^\circ\tan 80^\circ}$

$=\tan(110^\circ-80^\circ)$

$=\tan 30^\circ$

$=\frac{1}{\sqrt{3}}$.

1 — $\tan(110^\circ-80^\circ)$

1 — $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{2}$

Mostly done well.

(c) $\cos\frac{\pi}{12}=\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$

$=\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$=\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\times\frac{1}{2}$

$=\frac{\sqrt{6}+\sqrt{2}}{4}$ = RHS.

1 — mark for $\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$

1 — $\frac{\sqrt{6}+\sqrt{2}}{4}$ with $\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\times\frac{1}{2}$ as a preceding step.

Any other method used then student got a mark however the second mark was only given if they could arrive to $\frac{\sqrt{6}+\sqrt{2}}{4}$ with correct working out.

Suggested solutions	Marker's Feedback

Q.(7)(a)(i) $\sqrt{3}\cos\theta - \sin\theta \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$.

(i) $R > 0$

Equating parts,

$\sqrt{3}\cos\theta = R\cos\theta\cos\alpha$ | $-\sin\theta = R\sin\theta\sin\alpha$

$\sqrt{3} = R\cos\alpha$ (1) | $-1 = R\sin\alpha$ (2)

(1)^2 + (2)^2:

$R^2(\cos^2\alpha + \sin^2\alpha) = (\sqrt{3})^2 + (-1)^2$

$R = 2$. 1 3 ✓

Sub. $R = 2$ in (1)! Sub. $R = 2$ in (2)!

$\sqrt{3} = 2\cos\alpha$ $-1 = 2\sin\alpha$

$\frac{\sqrt{3}}{2} = \cos\alpha$. $-\frac{1}{2} = \sin\alpha$] 1

$\alpha = \frac{11\pi}{6}$. 1

$\therefore \sqrt{3}\cos\theta - \sin\theta \equiv 2\cos(\theta - \frac{11\pi}{6})$

(ii) Hence $\sqrt{3}\cos\theta - \sin\theta = 1$.

$2\cos(\theta - \frac{11\pi}{6}) = 1$.

$\cos(\theta - \frac{11\pi}{6}) = \frac{1}{2}$.

Noting $-\frac{11\pi}{6} \le \theta - \frac{11\pi}{6} \le \frac{\pi}{6}$. 2

$\theta - \frac{11\pi}{6} = -\frac{5\pi}{3}, -\frac{\pi}{3}$.

1

$\theta = \frac{\pi}{6}, \frac{3\pi}{2}$. 1

Some students couldn't find the correct value for α

Common errors:-

$\alpha = \frac{\pi}{6}$, $\alpha = \frac{5\pi}{6}$

* Mark deducted if they didn't justify α in 4th quadrant.

* For part (i), if they got wrong value for α and as a result the domain change made part (ii) question easier, then no carry forward mark given for finding θ.

* Some students didn't pay attention to domain when finding values of θ.

* Mark deducted if θ not given in radians.

Suggested solutions	Marker's Feedback

Q.(7)(b)

$3\sin\theta - 2\cos\theta \equiv R\sin(\theta - \alpha), R>0$ 1 for right pick

$3\sin\theta - 2\cos\theta \equiv R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

Equating pairs

$3\sin\theta = R\sin\theta\cos\alpha$ | $-2\cos\theta = -R\cos\theta\sin\alpha$

$3 = R\cos\alpha$ (1) | 2 1 for $= R\sin\alpha$ (2)

(1)^2 + (2)^2: $R^2(\cos^2\alpha + \sin^2\alpha) = 3^2 + 2^2$ ✓

$R = \sqrt{13}$ ✓

Sub. $R=\sqrt{13}$ in (1): | Sub. $R=\sqrt{13}$ in (2):

$3 = \sqrt{13}\cos\alpha$ | $2 = \sqrt{13}\sin\alpha$

$\frac{3}{\sqrt{13}} = \cos\alpha$ | $\frac{2}{\sqrt{13}} = \sin\alpha$

$\alpha = 33^\circ 41'$! 1 for α ✓

Hence solving $3\sin\theta - 2\cos\theta = 1$

$\sqrt{13}\sin(\theta - 33^\circ 41') = 1$.

$\sin(\theta - 33^\circ 41') = \frac{1}{\sqrt{13}}$ 1

$\theta - 33^\circ 41' = 16^\circ 6', 163^\circ 54'$.

$\theta = 49^\circ 47'$ 1 , $197^\circ 35'$.

* Some students didn't choose the right form $R\sin(\theta - \alpha)$
* Need to pay attention to $0 < \alpha < 90^\circ$
* A few students didn't list all possible solutions.
* Mark deducted when θ not given in degrees.

Suggested solutions	Marker's Feedback

Q.(8)(a) $3\sin\theta + 4\cos\theta = 2$

$\frac{3\times 2t}{1+t^2} + \frac{4(1-t^2)}{1+t^2} = 2.$

$6t + 4 - 4t^2 = 2 + 2t^2$

$0 = 6t^2 - 6t - 2$

$0 = 3t^2 - 3t - 1.$

$t = \frac{3 \pm \sqrt{(-3)^2 - 4\times 3\times -1}}{2\times 3}$

$= \frac{3 \pm \sqrt{21}}{6}.$

$\tan\left(\frac{\theta}{2}\right) = \frac{3+\sqrt{21}}{6}$ | $\tan\left(\frac{\theta}{2}\right) = \frac{3-\sqrt{21}}{6}$

$\frac{\theta}{2} = 51^\circ 39'$ | $\frac{\theta}{2} = 165^\circ 9'$

$\theta = 103^\circ 18'$ | $\theta = 330^\circ 27'$

Test: $\theta = 180^\circ$

$3\sin 180^\circ + 4\cos 180^\circ = -4 \neq 2.$

180° is not a solution

$\therefore$ Solutions are

$\theta = 103^\circ 18', 330^\circ 27'$ 5

Lots of wrong answers for the solution of quadratic.

Many students have difficulty in finding the correct solutions for $\tan\frac{\theta}{2} = \frac{3-\sqrt{21}}{6}$

Some forgot to Test for 180°

Suggested solutions	Marker's Feedback

Q.(8)(b)(i) $\sqrt{2}\cos x + \sqrt{2}\sin x \equiv R\sin(x+\alpha)$

$\sqrt{2}\cos x + \sqrt{2}\sin x \equiv R\sin x\cos\alpha + R\cos x\sin\alpha$.

Equating parts,

$\sqrt{2}\cos x = R\cos x\sin\alpha$ | $\sqrt{2}\sin x = R\sin x\cos\alpha$

$\sqrt{2} = R\sin\alpha$ (1) | $\sqrt{2} = R\cos\alpha$ (2)

$(1)^2 + (2)^2$: $R^2(\cos^2\alpha + \sin^2\alpha) = (\sqrt{2})^2 + (\sqrt{2})^2$

$R^2 = 4$.

$R = 2$.

sub. $R=2$ in (1): $\sqrt{2} = 2\sin\alpha$ | sub. $R=2$ in (2): $\frac{\sqrt{2}}{2} = \cos\alpha$.

$\frac{\sqrt{2}}{2} = \sin\alpha$

$\alpha = \frac{\pi}{4}$.

4

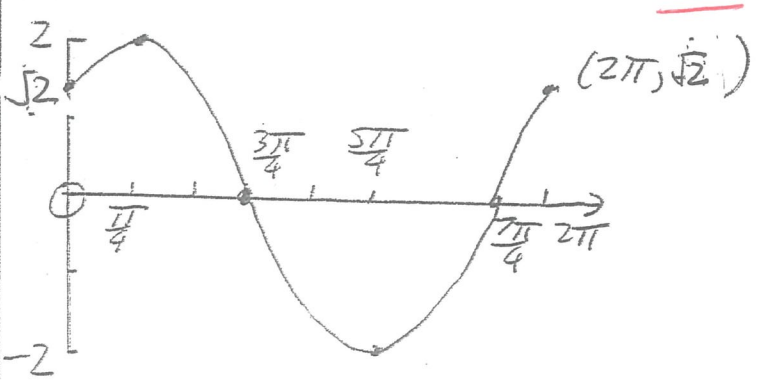
$\therefore \sqrt{2}\cos x + \sqrt{2}\sin x = 2\sin\left(x+\frac{\pi}{4}\right)$.

(ii) → Period $= 2\pi$. Amplitude $= 2$.

Phase $= \frac{\pi}{4}$.

$x = 0$ (or 2π) $y = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

3



Some wrote

$\tan\alpha = 1$

$\alpha = \frac{\pi}{4}$

They lost 1 mark.

Common mistakes

* wrong y intercept
* not writing all the x values

ie $0, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$

Suggested solutions	Marker's Feedback
Q.(9)(a) $\cos 2x - \sin x = \frac{1}{2}$ $(1-2\sin^2 x) - \sin x = \frac{1}{2}$ 1 $0 = 4\sin^2 x + 2\sin x - 1$. $\sin x = \frac{-2 \pm \sqrt{(-2)^2 - 4\times 4 \times -1}}{2\times 4}$ $= \frac{-2 \pm 2\sqrt{5}}{8}$ $= \frac{-1 \pm \sqrt{5}}{4}$ 1 4 If $\sin x = \frac{-1+\sqrt{5}}{4}$ $x = 18^\circ, 162^\circ$. 1 If $\sin x = \frac{-1-\sqrt{5}}{4}$ $x = 234^\circ, 306^\circ$. 1	* The vast majority of students knew to use $\cos 2x = 1 - 2\sin^2 x$ & get a quadratic equation using $\sin x$. * A very worrying group of about 10 students forgot the NULL FACTOR law when solving the equation. * Many students didn't realise $\frac{-1+\sqrt{5}}{4}$ and $\frac{-1-\sqrt{5}}{4}$ are (a) NOT EQUAL and (b) NOT OPPOSITES so that there are actually 2 sets of 2 solutions $[\sin^{-1}(\frac{-1+\sqrt{5}}{4}) = 18^\circ, 162^\circ$ & $\sin^{-1}(\frac{-1-\sqrt{5}}{4}) = 234^\circ, 306^\circ$.
(9)(b) (i) Either $\sin 8\theta + \sin 6\theta - \sin 4\theta - \sin 2\theta$ $= [\sin(7\theta+\theta) + \sin(7\theta-\theta) - [\sin(3\theta+\theta) + \sin(3\theta-\theta)]$ $= 2\sin 7\theta\cos\theta - 2\sin 3\theta\cos\theta$ 1 $= 2\cos\theta[\sin 7\theta - \sin 3\theta]$ $= 2\cos\theta[\sin(5\theta+2\theta) - \sin(5\theta-2\theta)]$ $= 2\cos\theta \times 2\cos 5\theta \sin 2\theta$ 1 2 $= 4\cos 5\theta \sin 2\theta \cos\theta$ = RHS QED.	(9)(b)(i) was mostly well done.
(or) $\sin 8\theta - \sin 4\theta + \sin 6\theta - \sin 2\theta$ $= \sin(6\theta+2\theta) - \sin(6\theta-2\theta) + \sin(4\theta+2\theta) - \sin(4\theta-2\theta)$ $= 2\cos 6\theta \sin 2\theta + 2\cos 4\theta \sin 2\theta$ 1 $= 2\sin 2\theta[\cos 6\theta + \cos 4\theta]$ $= 2\sin 2\theta[\cos(5\theta+\theta) + \cos(5\theta-\theta)]$ $= 2\sin 2\theta[2\cos 5\theta\cos\theta]$ 1 $= 4\cos 5\theta \sin 2\theta \cos\theta$.	(or) $\sin 8\theta - \sin 2\theta + \sin 6\theta - \sin 4\theta$ $= \sin(5\theta+3\theta) - \sin(5\theta-3\theta)$ $+ \sin(5\theta+\theta) - \sin(5\theta-\theta)$ $= 2\cos 5\theta\sin 3\theta + 2\cos 5\theta\sin\theta$ 1 $= 2\cos 5\theta[\sin 3\theta + \sin\theta]$ $= 2\cos 5\theta[\sin(2\theta+\theta) + \sin(2\theta-\theta)]$ $= 2\cos 5\theta \times 2\sin 2\theta\cos\theta$ 1 $= 4\cos 5\theta \sin 2\theta \cos\theta$.

Suggested solutions	Marker's Feedback

Q(9)(b)(ii) Hence solving

$\sin 8\theta + \sin 6\theta - \sin 4\theta - \sin 2\theta = 0$

$4\cos 5\theta \sin 2\theta \cos\theta = 0$ $\quad 0 \le \theta \le 2\pi$

Either $\cos 5\theta = 0$ $\quad 0 \le 5\theta \le 10\pi$

$5\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2},$

$\frac{11\pi}{2}, \frac{13\pi}{2}, \frac{15\pi}{2}, \frac{17\pi}{2}, \frac{19\pi}{2}$

$\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10},$

$\frac{11\pi}{10}, \frac{13\pi}{10}, \frac{3\pi}{2}, \frac{17\pi}{10}, \frac{19\pi}{10}$

$0 \le \theta \le 2\pi$

or $\sin 2\theta = 0$ $\quad 0 \le 5\theta \le 10\pi$. 3

$2\theta = 0, \pi, 2\pi, 3\pi, 4\pi$

$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \therefore 2\pi$

↑ Already in $\cos 5\theta = 0$ ↑

or $\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ → already in $\cos 5\theta = 0$.

Final solutions:

$\theta = 0, \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10}, \pi,$

$\frac{11\pi}{10}, \frac{13\pi}{10}, \frac{3\pi}{2}, \frac{17\pi}{10}, \frac{19\pi}{10}, 2\pi.$

A minority of students had no idea, but most students realised that there were 3 possibilities $\cos 5\theta = 0$, $\sin 2\theta = 0$ or $\cos\theta = 0$.

However, for $\cos 5\theta = 0$ MOST students forgot if $0 \le \theta \le 2\pi$ then $0 \le 5\theta \le 10\pi$ which meant they missed the solutions $\frac{7\pi}{10}, \frac{9\pi}{10}, \ldots$ to $\frac{19\pi}{10}$.

1 for $\cos 5\theta = 0$ or $\sin 2\theta = 0$ or $\cos\theta = 0$

1 for getting some of the answers

& 1 for ALL answers.

Suggested solutions	Marker's Feedback

Q.(10)(a) Step 1: Show true for $n=1$:

LHS $= 1\times 2 = 2$.

RHS $= \frac{1}{3}\times 1\times(1+1)(1+2) = 2$.

LHS $=$ RHS $\Rightarrow$ True for $n=1$.

Step 2: Assume true for $n=k$

i.e. $1\times 2 + 2\times 3 + \ldots + k(k+1) = \frac{1}{3}k(k+1)(k+2)$.

Step 3: Prove true for $n=k+1$

i.e. RTP $1\times 2 + \ldots + k(k+1) + (k+1)(k+2)$

$= \frac{1}{3}(k+1)(k+2)(k+3)$. ... 1

including statement

LHS

$1\times 2 + \ldots + k(k+1) + (k+1)(k+2)$] 1 for assumption

$= \frac{1}{3}k(k+1)(k+2) + (k+1)(k+2)$ (By assumption)

$= \frac{1}{3}k(k+1)(k+2) + \frac{1}{3}\times 3(k+1)(k+2)$

$= \frac{1}{3}(k+1)(k+2)(k+3)$

$=$ RHS QED 1 for finish. 4

If it is true for $n=k$ it is true for $n=k+1$.
Hence as it is true for $n=1$ it will be true for all positive integers n by the principle of mathematical induction.

Mostly done well.

(-1) mark if they did not mention 'by assumption' in question 10a only.

Suggested solutions	Marker's Feedback

Q.(10)(b) Step 1: Show true for $n=1$.

LHS
$= 5^2 - 3^2$
$= 16$
which is divisible by 16.

Step 2: Assume true for $n=k$
i.e. $5^{2k} - 3^{2k} = 16P$, P an integer.
or $3^{2k} = 5^{2k} - 16P$.

Step 3: Prove true for $n=k+1$.
i.e. RTP: $5^{2k+2} - 3^{2k+2} = 16Q$, Q an integer.

LHS
$= 5^{2k+2} - 3^{2k+2}$
$= 25 \times 5^{2k} - 9 \times (5^{2k} - 16P)$ (By assumption) — 1 for assumption.
$= 25 \times 5^{2k} - 9 \times 5^{2k} + 9 \times 16P$
$= 16 \times 5^{2k} + 9 \times 16P$
$= 16[5^{2k} + 9P]$
$= 16Q$, $Q = 5^{2k} + 9P$, an integer as k, P integer.

If it is true for $n=k$ it will be true for $n=k+1$. Hence as it is true for $n=1$ it will be true for all positive integers n by the principle of mathematical induction.

Mostly done well.

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Suggested solutions	Marker's Feedback

Q.(10)(c) Step 1: Show true for $n=1$.

LHS $= \frac{1}{1\times2\times3} = \frac{1}{6}$.

RHS $= \frac{1(1+3)}{4\times(1+1)(1+2)} = \frac{4}{24} = \frac{1}{6}$

LHS = RHS. True for $n=1$.

Step 2: Assume true for $n=k$

i.e. $\frac{1}{1\times2\times3} + .. + \frac{1}{k(k+1)(k+2)} = \frac{k(k+3)}{4(k+1)(k+2)}$

Step 3: Prove true for $n=k+1$

i.e. RTP

$$\frac{1}{1\times2\times3} + .. + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)} = \frac{(k+1)(k+4)}{4(k+2)(k+3)}$$ 1

$$= \underbrace{\frac{1}{1\times2\times3} + .. + \frac{1}{k(k+1)(k+2)}}_{\text{LHS}} + \frac{1}{(k+1)(k+2)(k+3)}$$]1

$$= \frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)} \text{ (By assumption)}$$]1 5

$$= \frac{k(k+3)^2 + 4}{4(k+1)(k+2)(k+3)}$$

$$= \frac{k^3+6k^2+9k+4}{4(k+1)(k+2)(k+3)}$$ ← Majority got to this step.

MUST SHOW HOW from assumption to finish

$$= \frac{k^3+k^2+5k^2+5k+4k+4}{4(k+1)(k+2)(k+3)}$$ 1

$$= \frac{k^2(k+1)+5k(k+1)+4(k+1)}{4(k+1)(k+2)(k+3)}$$ ⊛

$$= \frac{(k+1)(k^2+5k+4)}{4(k+1)(k+2)(k+3)}$$

$$= \frac{(k+1)(k+4)}{4(k+2)(k+3)}$$ 1

= RHS QED.

Note: Students could also use factor theorem & polynomial division here.

If it is true for $n=k$, it will be true for $n=k+1$. Hence as it is true for $n=1$ it will be true for all positive integers n by mathemical induction.

Then some students wrote the final step without any explanation and lost one mark. Others who did not finish only got 3.

Suggested solutions	Marker's Feedback

Q.(ii) (i) $\cos 4\theta$

$= \cos 2[2\theta)$

$= 2\cos^2 2\theta - 1$

$= 2[2\cos^2\theta - 1]^2 - 1$

$= 2[4\cos^4\theta - 4\cos^2\theta + 1] - 1$

$= 8\cos^4\theta - 8\cos^2\theta + 2 - 1$ 3

$= 8\cos^4\theta - 8\cos^2\theta + 1$

$=$ RHS QED.

Generally well done using various methods.

(ii) $\cos\frac{4\pi}{9} + \cos\frac{2\pi}{9}$

$A = \cos\left(\frac{\pi}{3} + \frac{\pi}{9}\right) + \cos\left(\frac{\pi}{3} - \frac{\pi}{9}\right)$

$= 2\cos\frac{\pi}{3}\cos\frac{\pi}{9}$ 2

[By $\cos(A+B) + \cos(A-B) = 2\cos A\cos B$.]

$= 2 \times \frac{1}{2} \times \cos\frac{\pi}{9}$

$= \cos\frac{\pi}{9}$.

$=$ RHS QED.

Generally well done.

(iii) Letting $\theta = \frac{\pi}{9}$, this means

$\cos 4\theta + \cos 2\theta = \cos\theta$

$[8\cos^4\theta - 8\cos^2\theta + 1] + [2\cos^2\theta - 1] = \cos\theta$.

$8\cos^4\theta - 6\cos^2\theta = \cos\theta$.

$8\cos^4\theta - 6\cos^2\theta - \cos\theta = 0$

$\therefore 8\cos^4\frac{\pi}{9} - 6\cos^2\frac{\pi}{9} - \cos\frac{\pi}{9} = 0$ 3

$\cos\frac{\pi}{9}[8\cos^3\frac{\pi}{9} - 6\cos\frac{\pi}{9} - 1] = 0$.

As $\cos\frac{\pi}{9} \neq 0$,

$8\cos^3\frac{\pi}{9} - 6\cos\frac{\pi}{9} - 1 = 0$

$\therefore \cos\frac{\pi}{9}$ is a solution to

$8x^3 - 6x - 1 = 0$.

Poorly done overall. Students must use (i) & (ii) to achieve any marks.

END OF SOLUTIONS!